

Matter in the Bulk and its Consequences on the Brane: A Possible Source of Dark Energy

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The usual brane world scenario with anti de Sitter bulk has been generalized by considering a general form of energy momentum tensor in the bulk. The modified Einstein equation on the brane has been constructed. Two examples have been cited of which, the first one shows the usual brane equations when matter in the bulk is a negative cosmological constant. In the second example, the bulk matter is in the form of perfect fluid and as a result, an effective perfect fluid is obtained in the brane. Also it is noted that the effect of the dust bulk on the brane shows a dark energy behaviour and may be a possible explanation of the dark energy from the present day observational point of view.

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Branes play a fundamental role in the string theory, especially the D-branes on which open strings can end. The open strings describing the non gravitational sector are supposed to be attached to the branes at their end points, while the closed strings on the gravitational sector can propagate in the bulk, which is the higher dimensional embedding spacetime [1]. However, drastic simplifications are usually made regarding the localization of matter field on the brane with the aim to establish consistency of the observational predictions. According to the standard brane world idea, the particles are confined on a hypersurface known to be the brane, embedded in a higher dimensional anti de-Sitter spacetime called the bulk. The scalar field [2] has further been included in the bulk in order to generalize the so called Randall-Sundrum model [3].

On the other hand, from the phenomenological point of view, the matter in the bulk can be anything and is supported by the existence of a higher dimensional embedding space. The existence of bulk matter may modify the cosmological evolution on the brane to a great extent and may lead to possible explanation of the present day observations. It may be speculated that though the matter in the bulk [4] has no exotic features, yet its gravitational attributes in the brane is a major source of dark energy. Then, as a result of this bulk matter and Z_2 symmetry, the divergence of the brane energy-momentum tensor is no longer zero [5, 6]. This non conservation of brane energy-momentum tensor reflects an exchange of energy momentum between the bulk and the brane. In case of vacuum or (negative) cosmological constant in the bulk, there is no such exchange and hence the Bianchi identity simply imposes a constraint on

the projected Weyl tensor on the brane [7, 8]. In this letter, we attempt to outline the procedure for arriving at the final form of the field equations in the brane along with the effect induced by the bulk containing the most general form of matter. This effort will hopefully widen the scope of studying the brane world models in a more general context.

Suppose the five dimensional metric on the bulk is written as

$$ds^2 = g_{AB} dy^A dy^B \quad (1)$$

and the brane is defined to be the hypersurface $y^5 = \text{constant}$, (say zero). So $n_A = \delta_A^5$ is the unit normal space like vector on the brane hypersurface. Assuming $g_{55} = 1$ [5], the induced metric on the brane can be written as

$$q_{AB} = g_{AB} - n_A n_B \quad (2)$$

Thus a natural choice of coordinates is $y^A = (x^\mu, y^5)$ with $x^\mu = (t, x^i)$ as the space-time coordinates on the brane. The extrinsic curvature orthogonal to n^A is given by

$$K_{AB} = \frac{1}{2} \mathcal{L}_{\vec{n}} g_{AB}$$

Now, using the five dimensional Einstein equation on the bulk

$$G_{AB} = \kappa_5^2 [T_{AB} + S_{AB} \delta(y^5)] \quad (3)$$

the Einstein equation on the brane can be written as [5, 6]

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$$G_{\mu\nu} = \frac{2}{3} \kappa_5^2 \left[T_{AB} q_\mu^A q_\nu^B + \left(T_{AB} n^A n^B - \frac{1}{4} T \right) q_{\mu\nu} \right] - E_{\mu\nu} + K K_{\mu\nu} - K_\mu^\sigma K_{\nu\sigma} - \frac{1}{2} q_{\mu\nu} (K^2 - K^{\alpha\beta} K_{\alpha\beta}) \quad (4)$$

where T_{AB} is the energy-momentum tensor on the bulk, $S_{\mu\nu}$ is the effective energy-momentum tensor localized on the brane and $\delta(y^5)$ stands for the localization of brane contribution.

Also, $E_{\mu\nu} = C_{BDF}^A n_A n^D q_\mu^B q_\nu^F$, is the projection of the bulk Weyl tensor orthogonal to n^A , with $E_{[\alpha,\beta]} = 0 = E_\alpha^\alpha$. The equation (4) in fact follows from the Gauss equation and the decomposition of the Riemann tensor into Weyl curvature, the Ricci tensor and the Ricci scalar.

Also the Codazzi equation has the form

$$\nabla_\nu K_\mu^\nu - \nabla_\mu K = \kappa_5^2 T_{AB} n^A q_\mu^B \quad (5)$$

Further, remembering Israel's junction condition [5,6,8,9] on the brane hypersurface (assuming the brane to be infinitely thin), we get

$$[q_{\mu\nu}] = 0 \quad \text{and} \quad [K_{\mu\nu}] = -\kappa_5^2 \left(S_{\mu\nu} - \frac{1}{3} S q_{\mu\nu} \right) \quad (6)$$

$$\text{where } [f] = \lim_{y^5 \rightarrow 0^+} f - \lim_{y^5 \rightarrow 0^-} f = f^+ - f^-.$$

Now imposing Z_2 symmetry on the bulk spacetime with brane as the fixed point we have

$$K_{\mu\nu}^+ = -K_{\mu\nu}^- = -\frac{1}{2} \kappa_5^2 \left(S_{\mu\nu} - \frac{1}{3} S q_{\mu\nu} \right) \quad (7)$$

(henceforth neglecting the indices \pm for simplicity)

Then from equation (4) using (7), the form of the effective Einstein equation on the brane takes the form

$$G_{\mu\nu} = \kappa_4^2 \tau_{\mu\nu} + \kappa_5^4 \Pi_{\mu\nu} - E_{\mu\nu} + T_{\mu\nu}^{(P)} \quad (8)$$

where

$$T_{\mu\nu}^{(P)} = \frac{2}{3} \kappa_5^2 \left[T_{AB} q_\mu^A q_\nu^B + \left(T_{AB} n^A n^B - \frac{1}{4} T \right) q_{\mu\nu} - \frac{1}{8} \kappa_5^2 \lambda^2 q_{\mu\nu} \right] \quad (9)$$

is termed as the projected part of the bulk energy-momentum tensor on the brane.

Here we have decomposed $S_{\mu\nu}$ as

$$S_{\mu\nu} = -\lambda q_{\mu\nu} + \tau_{\mu\nu} \quad (10)$$

with $\lambda (> 0)$ and $\tau_{\mu\nu}$, the vacuum energy (brane tension) and the energy-momentum tensor respectively in the brane. The $\Pi_{\mu\nu}$ term, which is quadratic in $\tau_{\mu\nu}$, has the usual expression [5, 8]

$$\Pi_{\mu\nu} = \frac{1}{12} \tau \tau_{\mu\nu} - \frac{1}{4} \tau_{\mu\alpha} \tau_\nu^\alpha + \frac{1}{24} q_{\mu\nu} [3 \tau_{\alpha\beta} \tau^{\alpha\beta} - \tau^2] \quad (11)$$

and $\kappa_4^2 = \frac{1}{6} \kappa_5^4 \lambda$.

As a consequence of the Codazzi equation and the Z_2 symmetry, the divergence of the brane energy-momentum tensor is related to the projection of the bulk energy-momentum tensor by the relation [6]

$$\nabla_\nu \tau_\mu^\nu = -2 T_{AB} n^A q_\mu^B \quad (12)$$

Thus equation (8) is the effective Einstein equation on the brane with matters in the bulk and brane, characterized by the energy-momentum tensors T_{AB} and $\tau_{\mu\nu}$ respectively. Note that, these matters are not completely arbitrary but are restricted by the relation (12) [Also the Bianchi identity $\nabla_\nu G_\mu^\nu = 0$ gives one more restriction relating the divergence of $E_{\mu\nu}$, $\Pi_{\mu\nu}$ and T_{AB} and is complicated in nature].

We shall now cite below a few examples as special cases:

I. Bulk as the anti de-Sitter space time :

Here the energy-momentum tensor in the bulk is

$$T_{AB} = -\Lambda g_{AB} \quad (13)$$

Λ being the five dimensional cosmological constant.

Now using (13), the effective Einstein equation (8) simplifies to

$$G_{\mu\nu} = \kappa_4^2 \tau_{\mu\nu} + \kappa_5^4 \Pi_{\mu\nu} - E_{\mu\nu} - \Lambda_{(4)} q_{\mu\nu} \quad (14)$$

where $\Lambda_{(4)} = \frac{1}{2} \kappa_5^2 \left(\Lambda + \kappa_5^2 \frac{\lambda^2}{6} \right)$ is the effective cosmological constant on the brane [5, 6, 8].

This is the usual Randall-Sundrum brane model.

II. Bulk matter is in the form of perfect fluid :

Here the form of the energy-momentum tensor is

$$T_{AB} = (\rho + p) u_A u_B + p g_{AB} \quad (15)$$

where the velocity five vector is given by u_A and ρ , p stand for the density and pressure of the perfect fluid in the bulk. Here u_A is the time like vector orthogonal to the space like normal vector n_A i.e, $u_A n^A = 0$.

Thus the projected part of the bulk matter (given by equation (15)) given in equation (9) simplifies to

$$T_{\mu\nu}^{(P)} = \frac{2}{3} \kappa_5^2 [(\rho_{eff} + p_{eff}) u_\mu u_\nu + p_{eff} q_{\mu\nu}] \quad (16)$$

where the effective density ρ_{eff} and the effective pressure p_{eff} on the brane due to the projection of the bulk energy-momentum tensor may be written as

$$\rho_{eff} = \frac{3\rho}{4} + \frac{\kappa_5^2 \lambda^2}{8}, \quad p_{eff} = p + \frac{\rho}{4} - \frac{\kappa_5^2 \lambda^2}{8} \quad (17)$$

We note from the above expressions (17) that the effective density and pressure on the brane contributed by the bulk matter are combinations of

the bulk energy density, bulk pressure and the brane tension. An interesting conclusion arising from the above consideration is that, a pure dust in the 5-D bulk contributes to both density and pressure of the brane in the form of an effective perfect fluid. All these contributions from the bulk add to the already existing brane energy density and pressure arising out of the brane matter itself. Now

$$\rho_{eff} + p_{eff} = \rho + p$$

$$\text{and} \quad \rho_{eff} + 3p_{eff} = \frac{3\rho}{2} + 3p - \frac{\kappa_5^2 \lambda^2}{4}.$$

Note that for dust filled bulk, if $\rho < \frac{\kappa_5^2 \lambda^2}{6}$, then strong energy condition is violated for the induced matter in the brane. But for perfect fluid in the bulk, if the dominant energy condition is satisfied, then it will also be obeyed by the induced matter in the brane. In other words, bulk with dust matter shows its impact on the brane models as a possible source of dark energy. Therefore, although we can not probe the bulk, but the cosmological evolution on the brane is modified by the dark energy through the bulk gravitational influence. So our scenario of brane dynamics drastically changes if we include the above additional contributions, which might be an interesting problem for future work.

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